# *Geometric Discovery Through Interactive Software*

An Honors Thesis (HONRS 499)

by

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# Abstract


The National Council of Teachers of Mathematics (NCTM) has recently called for many changes within mathematics education. The NCTM *Curriculum and Evaluation Standards for School Mathematics* develops the basis for these changes. A major theme throughout the *Standards* is that learning should be accomplished by active experimentation, rather than passive absorption. One method for implementing such a change is the use of computers in the teaching of mathematics. Many software packages exist to aid in teaching such high school subjects as algebra and pre-calculus, however, there are presently few packages on the market that specialize in geometry, and even fewer resources to aid teachers in utilizing this software. *The Geometer's Sketchpad* and *Cabri-Géomètre* are the most powerful computer tools available for geometric exploration. They allow students to modify particular geometric aspects of a figure and instantly watch changes caused by those modifications. Through observations, students are then able to make educated conjectures about geometric ideas. This project branched in two directions. The first was the comparison of the two interactive software packages listed above. The second was the development of computer laboratory activities to guide students in their discovery of geometric concepts.

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# *Implementing the NCTM Standards in Geometry through Technology*

# INTRODUCTION

Traditionally, teachers have taught geometry through passive methodologies. The teacher may draw a diagram on the board and explain with eloquent formal reasoning the basis for accepting the validity of a statement concerning the diagram, but many students watch in awe as their teacher performs a seemingly magical mathematical process. In *Curriculum and Evaluation Standards for School Mathematics*, commonly referred to as simply the *Standards*, the National Council of Teachers of Mathematics (NCTM) calls for decreased usage of such a teaching style and an increased emphasis on students playing an active role in the learning process (NCTM 1989, 10). This approach to teaching geometry entices students to discover geometry in a way similar to ancient geometers, such as Euclid and Pythagoras, and to modern geometers.

The word geometry comes from a Greek word literally meaning to measure land. It is with this purpose that the study of geometry began. As humans attempted to measure, they began to notice relationships between points, lines, and planes–the primitive, undefined terms of geometry. As they continued to notice relationships they began to observe, experiment with, conjecture about, and prove such relationships. From these beginnings, geometry became the formalized system that is taught in schools today. By allowing students to work through and develop the concepts of geometry through their own observations, experiments, conjectures, and proofs, teachers can present geometry as a dynamic and interesting field of study. Again, the *Standards* express this same idea (NCTM 1989, 5).

The main focus of this paper is the incorporation of technology into geometry education. However, technology is not the only means of learning or investigating. Traditionally manipulatives, pencil and paper diagrams, and real world examples have been used in the teaching of geometry. It is not the intent of the author to downplay the importance and usefulness of these learning methodologies. Instead, students need the chance to work with traditional as well as technological methodologies. It is also a necessity that students learn which approach is most beneficial for the problem at hand.

## THE SCIENCE OF PATTERNS

Accomplishing this change in geometry education requires that students develop their abilities in observing patterns. The development of skills in pattern observation is a goal that is especially prevalent in the *Standards* given for elementary and middle schools. Although not limited to these levels, the development of observation skills needs to occur early in one's mathematical development. Lynn Arthur Steen concisely refers to mathematics as "the science of patterns" (611). If students see the underlying patterns of mathematics, they will begin to develop a true understanding of mathematical thought. Mathematics is generally viewed as a set of rules that must be memorized, but by seeing and working with patterns and developing ideas through reasoning, students become aware that mathematics m akes logical sense (NCTM 1989, 29).

## GEOMETRIC VISUALIZATION

By its very nature, geometry is a visual area of study. To develop students' understanding of geometry, pedagogy must incorporate learning and teaching strategies which incorporate visualization. To explain this concept further, it is beneficial to look at an example. Many high school geometry students have at one point probably accepted the validity of the following statement.

> *If two sides and the non-included angle of one triangle are congruent to the corresponding parts of a another triangle, then the triangles are congruent.*

On initial investigation, such a statement appears to be true even after looking at a simple diagram. But utilizing visualization skills, one can stretch and shrink parts of the triangle while retaining the correspondence above, to demonstrate that there is at least one triangle with the above correspondence that is not congruent to the original triangle. [See Figure 1.]

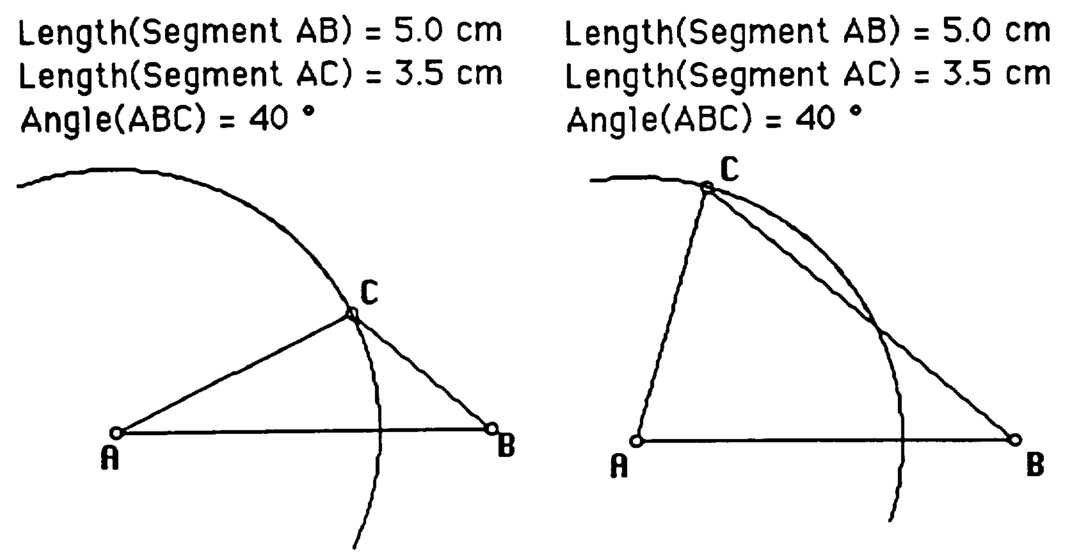



*Figure 1: Using a computer software package to model a geometric situation where two sides and the non-included angle of one triangle are congruent to the corresponding parts of another triangle, students can visually produce an example where the two triangles are not congruent.*

Visualization is a tool by which students gain deeper understanding of mathematical concepts through visual representations. Walter Zimmerman, Professor of Mathematics at the University of the Pacific, and Steve Cunningham, Professor of Computer Science at California State University Stanislaus, are the editors of an anthology published by the Mathematical Association of America entitled *Visualization in Learning and Teaching Mathematics*. The editors point out:

> Mathematical visualization is not "math appreciation through pictures." The intuition which mathematical visualization seeks is not a vague kind of intuition, a superficial substitute for understanding, but the kind of intuition which penetrates to the heart of an idea. It gives depth and meaning to understanding, serves as a reliable guide to problem solving, and inspires creative discoveries. (4)

Dina van Hiele and Pierre van Hiele conducted research into the thought processes needed for learning geometry. In their work, they consider the most basic level of geometric thought to be visualization. The next level is analysis. The combination of these two levels encompass the idea of visualization presented earlier. Their belief is that geometric learning at higher levels such as deduction cannot occur unless students have developed skills in visualization and analysis (Crowley 1-4).

## TECHNOLOGY

Society is undergoing a technological explosion. Technology is affecting every area of our lives from television to grocery shopping to education. With this thought in mind, educators must harness the available power of technology and use it to the advantage of their students. The *Standards* calls for increased use of computers in the study of mathematics by saying that computers are tools for "learning and doing mathematics" (NCTM 1989, 129). In recent years, mathematics educators have incorporated technology into their classrooms via tutorial programs, spreadsheets, computer algebra systems, and geometry software. This increase in the use of computers is due to technological advancements such as increased availability of computer hardware and the production of pedagogical software.

A look to the past reveals the beginnings of the computerization of geometric education in software such as *Logo* and *The Geometric Supposer*. Both packages utilize graphical environments to display geometric figures. At the time when these systems were created, many computers were textually oriented and command driven. In other words, the graphical environments of these systems were excellent when considered in light of the computer systems available.

*Logo* is an easy to learn and use programming language which allows students to graphically experiment with geometric ideas as they maneuver the *turtle* with geometric commands. *Logo* presently exists in many versions.

*The Geometric Supposer* was developed from 1985-1988 by Judah L. Schwartz and Michal Yerushalmy both under the direction of the Education Development Center. The package is published and distributed by Sunburst Communications. *The Geometric Supposer* is actually three separate packages. Each package is intended for the experimentation of one geometric shape–circles, triangles, or quadrilaterals. Within a given package, students are allowed to execute various Euclidean constructions, make measurements, and perform calculations. These constructions, measurements, and calculations are recorded and can be repeated for many other similar drawings. Through this repetition, students can make conjectures and test their validity on numerous diagrams.

Michal Yerushalmy and Richard Houde emphasize the idea that *The Geometric Supposer* allows students to "create geometry." She advocates a classroom where students participate in the learning of geometry through discussion and experimentation as opposed to simply watching or listening to the teacher explain ideas and concepts in geometry (Yerushalmy and Houde, 418). This idea is reminiscent of the beliefs portrayed in the *Standards.*

## DYNAMIC GEOMETRY SYSTEMS

*Logo* and *The Geometric Supposer* are still used to aid learning in many high schools and middle schools across the nation. Within the past few years, however, a new wave of geometric software, which owes its existence to the likes of *Logo* and *The Geometric Supposer*, has entered into the educational world. New computer systems, which are readily available, create graphical environments where the user interacts with the computer through visual means such as using a mouse to point an arrow at a menu command. Because of the visual aspects of geometry, these graphical environments are beneficial in increasing potential in geometry education and discovery.

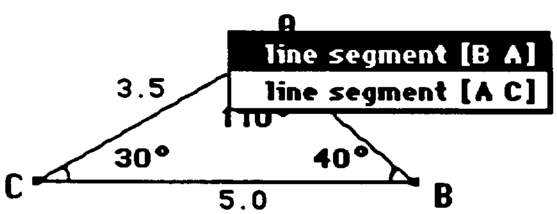
These geometry systems differ from *Logo* and *The Geometric Supposer* because they are dynamic, allowing continuous changes to figures. Previous software allowed only discrete changes in figures. A good analogy is seen in the difference between motion pictures and slide shows. A dynamic geometry system (DGS) offers an environment that allows in depth exploration with accuracy and efficiency.

Although there are more than two DGSs available, *The Geometer's Sketchpad* and *Cabri-Géomètre* are the most well-known. *The Geometer's Sketchpad* is available through Key Curriculum Press. It was created in 1991 as part of the "Visual Geometry Project" sponsored by the National Science Foundation and headed by Eugene Klotz of Swarthmore College and Dorris Schattschneider of Moravian College. *Cabri-Géomètre* was created by the French Laboratoire de Structures Discrète et de Didactique at IMAG (CRNS-UJF) in 1988.

Some important distinctions between the two software packages are highlighted below.

(1) **Selection of objects.** A difficult problem facing DGS users is the selection of objects for operation. For instance, if the user wants to create the line perpendicular to a segment through a certain point, the DGS must know what segment and what point it should utilize in the construction. *Cabri-Géomètre* utilizes a post-fixing method of selection. The user must select a command prior to the selection of objects. This allows the system to guide the user in the selection by displaying in words the object that is chosen. In the case of an ambiguity, a small menu appears to let the user specifically choose which object to select. [See Figure 2.] *Sketchpad*, on the other hand, utilizes a pre-fixing method of selection. The user must first select objects and then the command to perform. If the wrong types of objects are selected for a given command, then that command is not available for the user. However, an on-screen reference system does exist to alert the user of the correct objects needed to perform each command. In the case of ambiguity in selections, *Sketchpad* cycles through each of the objects when the user clicks the mouse button. Often it is still not clear which object has been selected.

*Figure 2: The ambiguity menu of Cabri-Géomètre.*



(2) **Measurement System.** Although both packages support systems of measurement, the application of these systems is quite different. In *Cabri-Géomètre*, if an object is measured, the measurement is attached to the object and changes dynamically with the object. [See Figure 3a.] *Sketchpad*, on the other hand, treats measurements as separate data which dynamically change with the object, but whose position is not reliant on the object itself. In other words, if the user modifies an object, associated measurements are instantly updated, but their position on the screen remains the same. [See Figure 3b.]

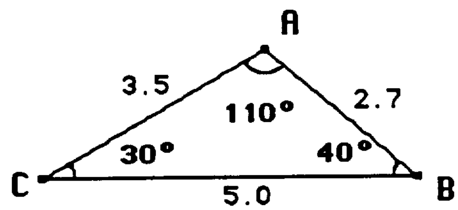
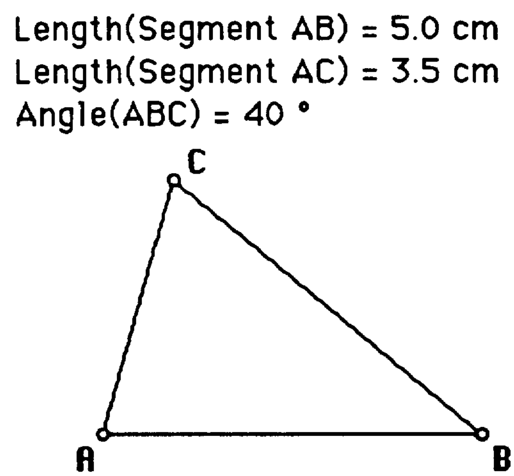



*Figure 3a: Cabri-Géomètre displays measurements as text directly attached to a figure.*

*Figure 3b: The Geometer's Sketchpad displays measurements as text unattached to the figure.*




(3) **Calculation system.** To provide a useful tool to students within the environment of a DGS, an arithmetic calculation system is beneficial. *Cabri-Géomètre* does not possess such a calculation system. *Sketchpad*, however, features an on-line scientific calculator at the touch of a mouse. This calculator can use previous measurements and calculations as input for new calculations. All calculations are dynamic and update as the related figure is modified. [See Figure 4.]

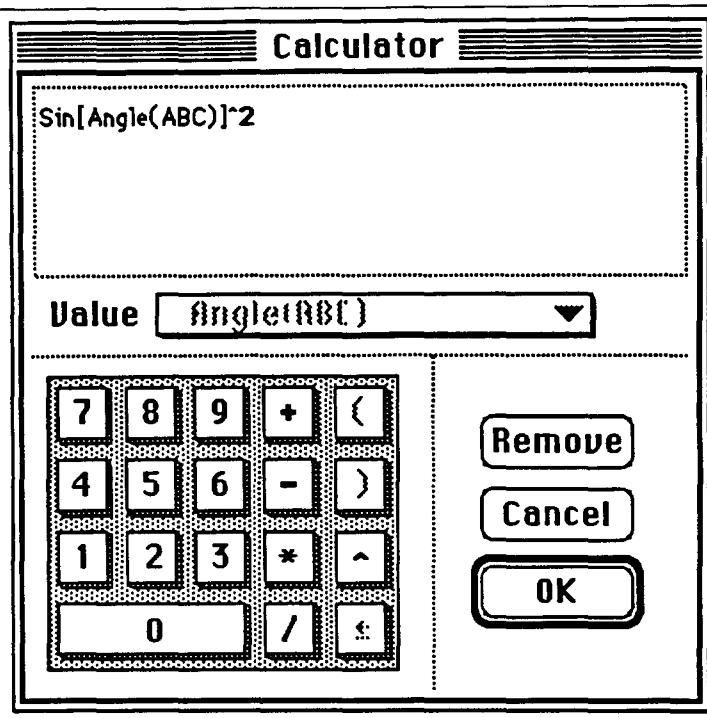



*Figure 4: The Geometer's Sketchpad offers a calculation system that can import previous measurements and calculations.*

(4) **Tools for teaching geometric transformations.** The *Standards* addresses the importance of transformations within geometry education. The only transformation explicitly supported by *Cabri-Géomètre* is the reflection of a point across a line. Through the combination of reflections through various lines, other transformations are possible, but often difficult to create. *Sketchpad* allows translations, reflections, rotations, and dilations directly as menu choices.

(5) **Inclusion of comments.** In the development of sketches, both teachers and students may find it helpful to include comments for clarification and reference. Comments on a sketch in *Cabri-Géomètre* appear in a window separate from the sketch. *Sketchpad* offers the ability to include comments directly on the sketch.

For more complete comparisons of these two packages see Habegger and Emert [1993] and DeTurck [1993]. A brief comparative overview of these two systems is given in figure 5. As is obvious, both systems offer similar capabilities as well as unique characteristics. It is hoped that in the future, DGSs will incorporate ideas from existing systems to develop easier to use and more powerful systems.

## USING DYNAMIC GEOMETRY SYSTEMS IN THE CLASSROOM

> Calculators and computers with appropriate software transform the mathematics classroom into a laboratory much like the environment in many science classes, where students use technology to investigate, conjecture, and verify their findings. In this setting, the teacher encourages experimentation and provides opportunities for students to summarize ideas and establish connections with previously studied topics. (NCTM 1989, 128)

Despite the power and usefulness of a DGS, learning will not occur without proper planning and guidance in the use of the system. Ideas of the *Standards* need to be implemented through prepared lessons and activities that implement the power of a DGS.

In the development of lessons that utilize a DGS, teachers should focus on activities that lead students to learning. Instead of telling students what they should see or prove, allow them to investigate ideas for themselves and draw their own conclusions.

Communication, both verbal and written, should also be incorporated into activities. Cooperative learning activities are excellent for fostering communication. In such a setting, students must rely on the work of the members of their group and must verbally share, compare, and discuss the subject matter. In addition, students should write about their experiences, discoveries and conjectures. Communication forces students to clarify and organize information prior to discussing it. This allows them to discover their own misunderstandings and gaps in knowledge.

Often students ask the question, "Where am I ever going to use this?" In keeping with the *Standards*, activities should not only place an emphasis on application but ideally should evolve from actual applied problems in everyday life (NCTM 1989, 9-10). The tendency is to concoct problems that appear to be from everyday life, but usually only implement physical objects into a simplified problem intended to give students practice on a certain operation or skill. Problems faced in life, often are complex and require the acquisition of more information and removal of excess information.

Many areas of mathematics are closely tied to geometry. Trigonometry, calculus, and algebra each have commonalities with geometry. Due to the compartmental structure of secondary education in America, students rarely have the opportunity to see the interconnections between the various courses taught.

## SOME EXAMPLE LABORATORY ACTIVITIES

In an effort to demonstrate the utilization of a DGS for the implementation of the ideas presented in the *Standards*, this project includes eight examples of geometry laboratory activities for use in high school classrooms that are equipped with *The Geometer's Sketchpad*. The goals that formulated the creation of these labs are based on the major ideas previously discussed: active participation by students, patterns, visualization, and technology.

The philosophy behind these labs is to lead students through a progression of thought whereby they experiment, make observations, and formulate conjectures. The emphasis is not on an initial presentation of a relationship and then having students see many examples that support its validity. Instead, students experiment with ideas, make note of relationships, and finally produce a rationale for accepting the validity of those relationships. Each lab serves as a lesson for student learning, not simply a demonstration or enrichment activity.

Although these labs are intended to aid students in learning geometry, this is not their only purpose. A secondary, but possibly more important, purpose is to develop students' abilities to learn mathematics. This is accomplished through the investigation of patterns and a visual approach to learning.

The labs, therefore, are structured with this philosophy in mind. This structure consists of three important elements: general objectives, icons, and guide sentences.

To give students a goal to attain and an understanding of where the activity is leading, each lab begins with a general learner objective. Students should not overlook the objectives of the labs, but should make sure that they understand them prior to beginning the lab. Giving students a solid framework of the task at hand will help them to organize information as it is obtained. The objectives are intentionally vague so that students cannot bypass discovery by simply reading the objectives. For instance, in *Determining a Circle* (Lab 2.1), the objective is stated as, "You will discover how many points determine a circle." This is opposed to stating the objective as, "You will discover that three non-collinear points determine a circle." Although the latter is our true objective, stating it in this manner would be inconsistent with the philosophy of these labs.

Each lab consists of a series of steps that are categorized by an associated icon. The icon alerts the students to the purpose of the step. An explanation of each of the icons appears in figure 6.

*Construct, label, measure* and *calculate* are all action icons. These icons alert students that some type of computer action is required for the step. Steps that require action by the student are summed up by a *guide sentence*. This sentence appears in bold, italic text. Immediately following it is usually a description of how to perform the action using *The Geometer's Sketchpad*. If there is no explanation, then it is assumed that the student knows how to perform the task at hand, either from past experience or by a description given previously in the lab.

The questions posed in a step marked by a *report* icon guide student thought and help them to organize their observations or conjectures. The students should formalize their responses to these questions via a written lab report. The report need not explicitly answer every question, but should provide an organized, thought out synopsis of the activities presented in the lab. Students may want to jot notes during the activity in response to the questions and then finalize and organize them in their final lab report.

The *note* icon does not require the student to take any action. Its purpose is to clarify information, provide definitions, and spur student insight.

# *Explanation of Icons*

| *ICON* | *NAME* | *MEANING* |
|---|---|---|
|  | CONSTRUCT | Construct geometric objects using the toolbox, the **Construct** menu, or the **Transform** menu. |
|  | LABEL | Use the label tool to label objects in the sketch. |
|  | MEASURE | Use the **Measure** menu to obtain measurements. |
|  | CALCULATE | Use the pull-down calculator and previous measurements to make calculations. |
|  | REPORT | Include the ideas from this section in your lab report. |
|  | NOTE | Read this information before proceeding with the lab; it will clarify unclear items. |

***Guide Sentences.*** A sentence that appears in bold and italics is called a guide sentence. It concisely states what you are to complete for this part of the lab. If further explanation is needed keep reading or refer back to a similar procedure.

*Figure 6: Explanation of icons utilized in each computer laboratory activity.*

The eight labs are organized into four sections that could be expanded at a future time. [See Figure 7.] The first section consists of two labs that introduce students to both geometry and *The Geometer's Sketchpad*. The second section consists of two labs that attempt to reveal the relationship between circles and perpendicular bisectors. The third section consists of one lab that develops a connection between trigonometry and geometry. Finally, the fourth section consists of three labs in transformational geometry.

*Introductory Labs*
Diagonals of a Polygon (Lab 1.1)
Experimenting with Triangles (Lab 1.2)

*Circles and Perpendicular Bisectors*
Determining a Circle (Lab 2.1)
Perpendicular Bisector (Lab 2.2)

*Trigonometry and Geometry*
Angle of Inclination (Lab 3.1)

*Transformational Geometry*
Reflections (Lab 4.1)
Translations (Lab 4.2)
Rotations (Lab 4.3)

*Figure 7: listing of sample laboratory activities included in Appendix A.*

# *Works Cited*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# *Appendix A*

# *Computer Laboratory Activities*

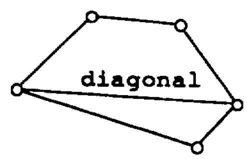
# *Diagonals of a Polygon*

Lab 1.1

**Objective:** You will discover a relationship between the number of sides of a polygon and the number of diagonals it has.

*A polygon is a shape created by connecting three or more points with line segments such that each point is the endpoint of exactly two segments and the segments intersect only at these points. The points are called the vertices of the polygon.*

*A diagonal of a polygon is a line segment that connects two non-adjacent vertices of a polygon.*



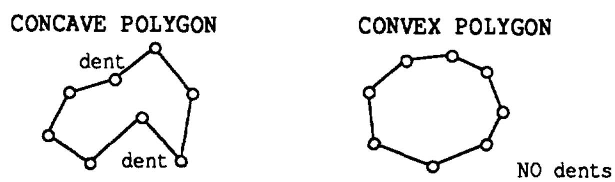

***Mark points for the vertices of a quadrilateral.*** A quadrilateral is a four-sided polygon. Select the *point tool* from the toolbox. When the *point tool* is active, clicking the mouse in the sketch will mark a point. To mark the four vertices of your quadrilateral, click the mouse at four different places in the sketch.

***Connect the vertices of the quadrilateral with line segments.*** To construct a line segment, you need to select two points so *Sketchpad* knows exactly which line segment to draw. To select objects, you must choose the *selection arrow* from the tool box, then clicking on the object with the *selection arrow* will select the object. It is important to note that selecting an object will deselect all other objects. This causes a problem, however, because some constructions require the selection of more than one object. To select more than one object, you must first select an object as above, then hold down the *shift key* while clicking on other objects. After you get accustomed to selecting objects, select two points that will form a side of your quadrilateral and choose from the **Construction** menu the **Segment** command. The two points will now be joined by a line segment. Select other pairs of points and construct segments between them. Make sure you have a quadrilateral before moving on to the next step.

How many line segments can you construct that connect non-adjacent corners of your quadrilateral. Would you expect the same to be true for all quadrilaterals? Explain your answer.

***Construct a pentagon (a five-sided polygon).*** Use a procedure similar to what you used to construct the quadrilateral. Mark five points and connect them with the segment command.

*When constructing figures for the rest of this project, it is best to make figures without dents. Figures without dents are called convex polygons and figures with dents are called concave polygons. An easy test to determine which type of polygon you have constructed is to look at the diagonals of the polygon. If any diagonal lies outside the polygon then the polygon is concave. If* ***all*** *of the diagonals lie inside the polygon, then it is convex If your polygon is concave, drag any point that makes a dent until the dent is gone.*




***Draw all of the diagonals of your pentagon.*** Select the *selection arrow* from the toolbox. Instead of drawing line segments between two adjacent vertices as you did to construct the pentagon, draw line segments between two non-adjacent vertices.

How many diagonals are in your pentagon? Would you expect the same to be true for all pentagons? Explain your answer.

***Construct a hexagon (a six-sided polygon) and its diagonals.***

*To ensure that you have constructed all diagonals, count the number of diagonals at each vertex. The number of diagonals at a vertex should be the same for all vertices of that polygon.*

*A nice trick to help count the diagonals is to select a diagonal using the selection arrow and then make it invisible by choosing from the* **Display** *menu the command* **Hide Segment**. *This should prevent you from counting some diagonals more than once or some not at all. After you have hidden all of the diagonals it is easy to make them visible again by selecting from the* **Display** *menu* **Show All Hidden**.

Count the diagonals in your hexagon.

Continue constructing and counting diagonals for polygons with more sides. Organize your observations in some type of table or chart.

Looking at the number of diagonals in different polygons, can you see a pattern emerging? How many new diagonals are added to a figure by adding one side? This information could be added to your chart as well.

In words, express the number of diagonals for a figure with n sides.

Write an algebraic expression for the number of diagonals in a polygon with n sides.

# *Experimenting with Triangles*

Lab 1.2

**Objective:** You will experiment with triangles and make conjectures about their properties.

***Construct a triangle.*** Recall the process of marking points in the plane and then connecting them by use of the **Segment** command.

***Label the points of the triangle.*** Choose the *text tool* from the toolbox—it's the little hand. This hand will turn black whenever it is pointing at a geometric object, such as a point, line, or circle. If an object is not labeled, then clicking on it with the *text tool* will label it. If an object is labeled, then clicking on it with the *text tool* will hide the label. Make sure all of your points are labeled before going on.

***Change the labels of the points of the triangle.*** Put the *text tool* on one of the labels, not one of the points. A capital "A" will appear on the hand whenever you are pointing at a label. Double click on the label and a box will appear in the middle of the screen. This screen allows you to change the label. Simply, type the new label and click the **OK** button when you're done. The point will now have the new label.

***Reposition the label.*** If you don't like where a label is placed, point the finger at the label and drag it around. Notice that the label must stay close to its object.

***Label the sides of the triangle.*** With the *text tool* click on a line segment. Again the hand will turn black when it is pointing at a geometric object. Click the mouse button and the label will appear. Change the labels to whatever letters you want. Label the rest of your line segments. These labels can be moved just like labels for points.

*Points are typically labeled with capital letters, while straight objects (line segments, lines, and rays) are labeled with lower case letters.*

***Measure the lengths of the sides of the triangle.*** Most geometric measurements can be performed on *Sketchpad* by selecting the objects that are involved in the measurement and choosing the appropriate measurement command. To measure a line segment, first click on it with the *selection arrow*. Two dots will appear on the line to show that it is selected. Then, from the **Measure** menu choose **Length**. The length will appear on your sketch. Measure the remaining sides of the triangle in the same way.

***Select three points that determine an angle of the triangle.*** To measure an angle, you must select three points that determine the angle. The proper order for selecting these points is: one endpoint, the vertex, and the other endpoint. Using the *selection arrow*, mark an angle to measure by selecting three points that determine that angle in the order listed above. Remember that to select multiple objects you must hold down the *shift key* while clicking the mouse. It is important, however, to make sure that only the three points of the angle are highlighted.

*Whenever you are selecting multiple objects, it is best to click on the first object* ***without*** *the shift key so that all other objects will be deselected.*

***Measure the angles of the triangle.*** To get the measurement of the angle marked by your three selected points, from the **Measure** menu choose **Angle**. If **Angle** is not available from the menu, try the above steps again—you may have more or less objects highlighted than the required three points. Otherwise, the angle measure should appear on your sketch. Repeat this process for all of the angles in your triangle.

By dragging the points of the triangle, construct triangles with the following characteristics. Make and record observations about each type of triangle including drawings of the triangles. It may be necessary to settle for measurements that are "close" to the intended measurements. Your interest should be in the general shape of the triangles.

*equilateral:* all sides are equal in length
*isosceles:* at least two sides are equal in length
*scalene:* no sides are equal in length
*equiangular:* all angles are equal in measure
*acute:* all angles are less than 90°
*right:* one angle is a right angle (its measure is 90°)
*obtuse:* one angle is greater than 90°

What combinations of triangles can you make? What combinations are impossible to make? For example, can you make an isosceles right triangle? Can you make an obtuse equilateral triangle? Try other combinations and record your results. If you find a combination that is impossible to make, give a brief explanation why. Include drawings of your triangles.

*Sketchpad is equipped with its own calculator which can use previous measurements. By clicking on the actual text of the measurement with the selection arrow, you can select measurments that will be included with your calculator.*

***Calculate the sum of the measures of the angles in the triangle.*** Highlight the three angle measurements by clicking on them with the *selection arrow*. Remember to hold down the *shift key* to select multiple objects. Go to the **Measure** menu and choose **Calculate**. A large calculator will pop up on the screen. Under the display area on the calculator should be a small window labeled *value* with one of your measurements appearing. This is called the *value window* and it acts exactly like a menu, so hold down the mouse button when pointing at it to see your options. A menu of measurements, functions, and constants will appear. Select one angle measurement from the *value window*. It should appear in the display on your calculator. Click on the + key of the calculator. Select a different angle measure from the *value window* and press the + key of the calculator again. Finally select the last angle measure. The display should now show, in words, the sum of the angle measures. Press the **OK** button to calculate. The calculator will disappear and the numerical sum will appear on your sketch.

***Calculate the sum of the lengths of the sides of the triangle.*** The procedure is the same as above, but select the three segment lengths before pulling out the calculator.

Drag parts of the triangle around to form different triangles. How are the two sums you just calculated affected by different types of triangles? Is there a rule concerning either of the sums? Make some conjectures. Look for relationships between the measure of an angle and the length of the side that is opposite it. Don't look for a strict numerical ratio, instead just notice relative sizes or ratios between the pairs.

# *Determining a Circle*

Lab 2.1

**Objective:** You will discover how many points determine a circle.

***Mark a point in your sketch.*** Select the *point tool* from the toolbox. When the *point tool* is active, clicking the mouse in the sketch will mark a point.

***Label the point X.*** Select the *text tool* from the toolbox. If your point is not labeled, then clicking on the point just created will label it. If your point is labeled, then clicking on the point will hide the label. Make sure your point is labeled. The label probably will not be "X". To change the label, put the finger on the label, not the point. A capital "A" will appear on the hand indicating that you are pointing at the label. Double click on the label and a box will appear in the middle of the screen. Type **X** and click the **OK** button to change the label of the point.

*A circle is all the points in a plane that are a certain distance away from a point. The point is called the center and the distance is called the radius. The center is not a point on the circle.*

***Construct a circle through the point X.*** Select the *circle tool* from the toolbox. Choose a point for the center of the circle. Any point other than **X** will work. Hold down the mouse button at that point and drag out the circle. As the cross hair moves with the circle, place it on the point **X**. This ensures that **X** is a point on the circle. When the cross hair gets close enough to **X** it should lock onto it. Let go of the mouse. Presto! Now you have a circle through the point **X**.

Can you choose other circles that go through **X** but have different centers? How many circles are there that go through or contain the point **X**?

*Open a new sketch. From the* **File** *menu choose* **New Sketch** *or use the keyboard shortcut listed to the right of the command in the menu.*

***Mark two different points in your sketch.***

***Label the two points X and Y.***

***Construct a circle through the point X.***

Does the circle go through the point **Y** also? If it doesn't, are there any circles that go through both **X** and **Y**? If it does, are there any circles that go through **X** but not through **Y**? Test your hypothesis by dragging the center of the circle around.

***Construct more circles that go through both X and Y.***

What do you think is true about the centers of the circles that go through **X** and **Y**? How many circles are there that go through **X** and **Y**?

*Open a new sketch.*

***Mark three different points in your sketch.*** Make sure that the points do not lie on the same line. Also try to locate the points within an inch of each other.

***Label the points X, Y, and Z.***

***Construct a circle that goes through X.***

Does the circle go through all three points? If it doesn't, can you drag the center until it does go through all three points? If it does, can you drag the center so it doesn't go through all three points?

***Construct more circles that go through X, Y, and Z.***

How many circles are there that go through **X**, **Y**, and **Z**? Explain why this is so.

*Think of constructing all circles through **X** and **Y**, and all circles through **Y** and **Z**. Their centers produce two lines. In how many points can two lines intersect? Add the line that consists of the centers of all circles through **X** and **Z**. How does that line relate to the other two lines?*

It is possible for two lines not to intersect. If this happened with the lines discussed above, how many circles would there be through the three points? What would have to be true about **X**, **Y**, and **Z** if the lines didn't intersect?

# *Perpendicular Bisector*

Lab 2.2

**Objective:** You will discover properties of the perpendicular bisector of a line segment and create a method to construct one.

***Construct a line segment.*** Select the *segment tool* from the toolbox. When the *segment tool* is active, line segments can be drawn by choosing an end point, holding the mouse button, and dragging to the other endpoint. The endpoints may be points already on your sketch. If they aren't points on your sketch, then they will automatically be placed on your sketch when the segment is drawn. Choose a point in your sketch to begin with and drag out a line segment.

***Label the endpoints of your line segment.***

***Construct the midpoint of the line segment.*** Choose the *selection arrow* from the toolbox and select the line segment. Now from the **Construction** menu choose the command **Point At Midpoint**. This command will construct the midpoint of a line segment.

***Label the midpoint of the line segment.***

***Through the midpoint, construct a line that is perpendicular to the segment.*** Begin by selecting the segment and the midpoint. Remember that to select multiple objects, you must hold down the *shift key*. Be sure that only the midpoint and the line segment are selected. Choose from the **Construction** menu, **Perpendicular Line**. A line will be constructed through the midpoint that is perpendicular to the line segment.

*A line that goes through the midpoint of a segment and is perpendicular to the segment is called the perpendicular bisector of the line segment.*

***Mark a point on the perpendicular bisector that will move freely.*** Select the line by clicking on it with the selection arrow. From the **Construct** menu choose **Point On Object**. This places a point somewhere on the selected object. You can drag this point around, but it will always remain on the object and will never move the object.

***Label the new point A.***

Make some conjectures about the distances from **A** to each of the endpoints of the line segment.

***Measure the distance from point A to each endpoint.***

What do you think is true about the distances from **A** to the endpoints? Keep in mind that your measurements may not be perfect. Be sure to drag point **A** along the line to look at more than one specific situation.

*Using this information, you can construct the perpendicular bisector of a segment. You know that any point on the perpendicular bisector of a segment is the same distance from both endpoints of the line segment. To construct the perpendicular bisector, you need to construct two points, each of which is the same distance from the endpoints of the line segment. Think about the definition of a circle.*

Describe in words the method of construction that you would use.

# *Angle of Inclination*

Lab 3.1

**Objective:** You will discover the relationship between the tangent of an angle and the slope of a line segment, and you will graph the tangent function.

***Construct a horizontal line segment.*** Select the *segment tool* from the toolbox. When the *segment tool* is active, line segments can be drawn by choosing an end point, holding the mouse button, and dragging to the other endpoint. The endpoints may be points already on your sketch. If they aren't points on your sketch, then they will automatically be placed on your sketch when the segment is drawn. Holding the *shift key* while drawing a segment limits the slopes at which a line segment can be drawn. This makes it possible to draw a truly horizontal line.

***Label this segment "horizon."*** Select the *text tool* from the toolbox. If the segment is not labeled, then clicking on the segment will label it. If your segment is labeled then clicking on the segment will hide the label. To change the label, put the finger on the label, not the segment. A capital "A" will appear on the hand indicating that you are pointing at the label. Double click on the label and a dialogue box will appear in the middle of the screen. Type **HORIZON** and click the **OK** button.

*The word "horizon" will refer to this line segment through the remainder of this lab.*

***Construct a segment with one endpoint on the horizon and the other above the horizon.*** Select the *segment tool* from the tool box. Choose the first endpoint of the segment on the horizon. Place the cross-hair directly on the line segment and then drag out the segment.

***Label the endpoint that is above the horizon as A.***
***Label the endpoint of the line segment that is on the horizon as B.***
***Label the right endpoint of the horizon as C.***

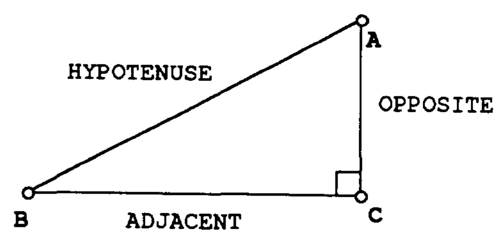
***Measure the slope of line segment AB.*** Select the *selection arrow* from the toolbox. To measure the slope of a line segment, highlight that segment making sure that no other object is selected. Click on the segment **without** pressing the *shift key*. This will deselect all objects while selecting the segment. Finally, from the **Measure** menu choose **Slope**.

*The angle of inclination of a line segment (or any straight object) is the angle from the horizon to the segment. Often people are concerned with angles of inclination when dealing with construction work and airplane navigation. In this case, the angle of inclination is angle ABC.*

***Measure the angle of inclination for segment AB.*** Choose the *selection arrow* from the toolbox. To measure an angle you must select the three points: an endpoint, the vertex, and the remaining endpoint. To select more than one object, you must hold down *shift* after selecting the first object. Select the appropriate points for angle **ABC** and from the **Measure** menu choose **Angle**. If **Angle** is not available from the menu then try this process again—you have more objects highlighted than the required three points. Otherwise, the measurement should appear on your sketch.

*There is a field of mathematics called trigonometry that is based on the study of right triangles. Look at the triangle below. The sides of this triangle are labeled with respect to angle **ABC**. In other words, segment **AC** is the side opposite angle **ABC** and segment **BC** is the side adjacent to angle **ABC**. The hypotenuse of a right triangle is always the side of the triangle opposite the right angle.*

*To calculate the tangent of an angle in a right triangle, divide the length of the side opposite the angle by the length of the side adjacent to the angle (not the hypotenuse). In the triangle on the right, the tangent of angle **ABC** is the length of **AC** divided by the length of **BC**.*



***Measure the length of segment AC and the length of segment BC.***

***Calculate the tangent of angle ABC.*** Choose the *selection arrow* from the toolbox. Highlight the measurements of segments **AC** and **BC** by clicking on the actual measurements in your sketch while holding down the *shift key*. From the **Measure** menu choose **Calculate**. The value window appears directly below the calculator display. The value window is actually a menu, so from the value window choose **Length(Segment AC)**. It will immediately appear on the display of the calculator. Use the calculator button for division which is **/**. Choose **Length(Segment BC)** from the value window. The final display should read:

**Length(Segment AC)/Length(Segment BC)**.

To complete the calculation click the **OK** button. The result will be placed on your sketch.

Compare the slope of **AB** to the tangent of its angle of inclination. What relationship do these two values have? Drag point **A** and see if this relationship appears to only be true sometimes.

Using this information make a conjecture about the relationship between the slope of **AB** and the tangent of its angle of inclination.

Systematically increase the angle of inclination. Keep a table of the angle measurements and their respective tangent values. Using this information draw a graph of the tangent function for angles between 0° and 180°. Let your horizontal axis be angle measures and your verticle axis be the corresponding tangent values.

# *Reflections*

Lab 4.1

**Objective:** You will experiment with and discover properties of a transformation of the plane known as a reflection.

Think about a mirror. When you look into a mirror, what happens to your left and right sides? What happens to words that you have printed on your clothes? What letters appear normally? What letters don't? Consider the distance of an object to the mirror. How does that affect the distance of its reflection from the mirror? Describe in detail the effects that a mirror has upon an object or a group of objects.

*Just as you described reflections in a mirror, geometrically you can describe reflections in a line.*

***Construct a line in the sketch.*** The *segment tool* can actually be exchanged for a tool which draws rays or a tool which draws lines. To change the tool, place the arrow on the *segment tool* and hold down the button. A set of tools will pop out to the left. Drag and highlight the far right tool. This is the *line tool.* Notice the arrows on both ends. Release the mouse button to select the *line tool.* Drag out a line just as if it were a line segment. The beginning place where you press the button will be marked by a point, and the place where you let go of the mouse will be marked by a point. These two points determine the line.

*Before moving on in the lab, it would be beneficial to switch back to the segment tool. To do this, hold down the mouse button on the line tool and highlight the first box—the segment tool.*

***Mark a point in the sketch that is not on the line.***

***Label the point P and the line m.***

Predict what would happen if the line actually was a mirror and the half-plane that doesn't contain **P** was the "world" that appears to exist within a mirror. Where would the reflection of the point appear on the other side of the line? Include a rough sketch of the point, line, and a guess at its reflection.

***Reflect the point through the line.*** The first step is to choose the line of reflection, which is called the mirror. Select the line **m** with the *selection arrow*, then from the **Transform** menu, choose **Mark Mirror "m"**. The label of the line selected always will appear inside the quotation marks. Now that you have marked the mirror, select the point **P** which you want to reflect through the mirror. Finally, from the **Transform** menu choose **Reflect**. **P** will be reflected through your mirror.

***Label the mirror image of your original point.*** Do not change the label of this point. The apostrophe attached at the end of a label means that the point is the mirror image of another point. In this case, the original point is **P**, and the reflected point is **P'**. This helps keep track of what points are images of other points.

*For the remainder of this lab, the "mirror image" of a point, or any other object, will be referred to as simply the image of a point.*

*There is nothing special about either half-plane determined by the line **m**. Points from either one can be reflected through the line.*

Consider the reflection of the point **P'** in line **m**. Where does it lie? What is the relationship between **P** and **P'** under the reflection?

Drag the point **P** around and notice how its image **P'** moves. Make conjectures about the location of the image of a point through a reflection. Test those conjectures. Make a concise set of instructions to describe the procedure for locating the image of any given point by its reflection in a line.

What happens if you reflect a point that is on the line of reflection? Why?

***Construct a triangle.***

***Label the points of the triangle.***

***Reflect the triangle through your line.*** Remember to mark the mirror first. When reflecting, you can select more than one point. All objects selected when a reflection is performed will be reflected. In this case, select the three vertices (points) and the three sides (line segments) of the triangle before reflecting. The entire triangle can then be reflected.

***Label the points of the image of the triangle.*** Again, for easy reference, do not change the names of these image points.

Drag parts of the triangle around and notice changes to the image. Drag one of the points of the triangle to the opposite side of the line of reflection. What happens? Does it change the shape or appearance of the triangle? Can a triangle lie on both sides of the line and still be reflected?

Look at a triangle and its image. What has changed because of the reflection? What has remained the same? Do the same properties hold true for other polygons and their images? Produce a list of properties that are preserved by a reflection. For instance, the length of a line segment is always preserved by a reflection. In addition, find at least one property that is not preserved by a reflection.

In your own words describe what happens when an object is reflected.

# *Translations*

Lab 4.2

**Objective:** You will apply your knowledge of reflections to experiment with and discover the properties of another transformation known as a translation.

***Draw a line.*** Use the *line tool* from the toolbox.

***Mark a point that is not on that line.***

***Construct a line parallel to the first line that passes through the point just marked.*** The command for constructing a line parallel to a given line through a given point requires the selection of a point and a line. Select both your line and point. Then, from the **Construct** menu choose **Parallel Line**.

***Label one of the parallel lines l and the other m.***

***Mark a point in your sketch that is not on either line.***

***Reflect a point through one of the lines.*** First, choose the line of reflection called the mirror. Select one of the parallel lines with the *selection arrow*, then from the **Transform** menu, choose **Mark Mirror "x"**. The label of the line selected will always appear inside the quotation marks, so depending on the line you selected, this command will appear differently on the screen. Now that you have marked the mirror, select the point just marked. Finally, from the **Transform** menu choose **Reflect**. The point will be reflected through your mirror.

***Reflect the image of the point through the other line.***

*A translation is the composite of two reflections through parallel lines. Therefore, the image of a point under a translation is the image created by the second reflection. Although the image of the first reflection is important to the creation of the image of the second reflection, it is disregarded in discussing translations.*

***Label the original point, the image under the first reflection, and the image under the second reflection.*** *Sketchpad* will label the image points with apostrophes to aid in recalling which point is an image of which point. It is best to leave these labels as they are.

Drag the original point around. What appears to be true about the three points? Do you think this will always be true? Explain.

***Construct a triangle.***

***Label the points of the triangle.***

***Reflect the entire triangle in one line then reflect its image through the other.***

*Since you have reflected an object through two parallel lines, you have translated the object. Under a translation, the image of the original triangle is the triangle created by the second reflection.*

***Label the points of the image of the triangle under the translation.***

***Measure the distance between the parallel lines.*** Recall that a pair of parallel lines is the same distance apart no matter where they are measured. The command to measure distance allows you to measure the distance between a point and a line or two points. Since you want to measure the distance between two parallel lines, you will have to select a point on one of the lines and select the other line. From the **Measure** menu, choose the **Distance** command. This will measure the distance between the two parallel lines.

***Measure the distance between the original point and its image under the translation.*** The **Distance** command can also measure the distance between two points. Select the two points and then **Measure** the **Distance**. Be sure that you are measuring the distance from the original point to the image under the *translation* which is the point created by the second reflection.

What relationship exists between the two distances just measured? Drag around the points of the original triangle and see if that relationship holds. With your knowledge of reflections, explain why this relationship should exist.

Recall properties that hold for reflections. Do these properties hold for translations? Do any properties that don't hold for one reflection, hold for translations?

In your own words, describe what occurs when an object is translated.

# *Rotations*

Lab 4.3

**Objective:** You will apply your knowledge of reflections to experiment with and discover the properties of another transformation known as a rotation.

***Draw two lines that intersect.*** Use the *line tool* from the toolbox. Remember to switch back to the segment tool when finished.

***Label the point of intersection of those two lines as -CENTER-.*** The hyphens before and after the word will make later measurements easier to read.

***Reflect a point that is not on either line through one of the lines.***

***Reflect the image from the above reflection through the other line.***

*Successive reflections through intersecting lines are called rotations.*

***Label the original point, the image under the first reflection, and the image under the second reflection.*** For convenience, *Sketchpad* will label the image points with apostrophes to aid in recalling which point is an image of which point. It is best to leave these labels as they are.

***Measure the distance from each of the three points of the rotation to the point -CENTER-.***

Drag the original point around. What appears to be true about the distance of the points to the **-CENTER-** ? Do you think this will always be true? Explain.

***Construct a triangle.***

***Label the points of the triangle.***

***Rotate the entire triangle around the -CENTER-.*** Reflect it through one of the lines, then reflect the image through the other line. Remember to select the whole triangle before performing the reflections.

*Since you have reflected an object through two intersecting lines, you have rotated the object. Under a rotation, the image of the original triangle is the triangle created by the second reflection.*

***Label the points of the image of the triangle under the rotation.***

***Measure the angle between the two intersecting lines.*** To measure the angle between two lines, you must select three points that determine that angle. You must select a point on one of the lines, the intersection of the two lines, and a point on the remaining line. The point of intersection must be the second point chosen. Then from the **Measure** menu choose **Angle**.

***Measure the angle from a point to -CENTER- to the image of the point under the entire rotation. Do this for all points of the triangle.***

What relationship exists between the measures of these angles and the measure of the angle between the intersecting lines? Drag around the original point and see if that relationship holds. With your knowledge of reflections, explain why this relationship exists.

Recall properties that hold for reflections. Do these properties hold for rotations? Do any properties that don't hold for one reflection, hold for rotations?

Describe in your own words what occurs when an object is rotated.

# *Appendix B*

Habegger, William V. and John W. Emert, "Cabri-Géomètre vs. The Geometer's Sketchpad: A Comparison of Two Dynamic Geometry Systems." *Notices of the American Mathematical Society* 40 (1993): 988-992.

# CABRI-GÉOMÈTRE VS. THE GEOMETER'S SKETCHPAD
## A comparison of two Dynamic Geometry Systems

*William V. Habegger and John W. Emert*

Historically, *Logo* and the *Geometric Supposers* were the most powerful and most frequently used software packages available for geometry education. Since technology and hardware capabilities have increased to their present state, these systems are no longer the most powerful available. Recently, two new packages have received much attention in North America—*Cabri-Géomètre* and *The Geometer's Sketchpad.* These dynamic geometry systems (DGS) differ from the previously mentioned packages because they harness the power of available computer hardware and offer a dynamic environment; that is, continuous modifications of particular geometric aspects within a figure will cause instant changes to the figure and its related measurements.

A DGS provides an exploratory, dynamic approach to geometry that simply cannot be supported by pencil and paper. While traditional manipulatives may be helpful in experimentation, their efficiency is questionable. It simply takes too long to create too few experiments, and these experiments often lack precision. In addition, DGS sketches provide the opportunity to view geometry as an exploratory activity as encouraged in the recent NCTM *Curriculum and Evaluation Standards for School Mathematics* [1].

*Cabri-Géomètre* was created by the French Laboratoire de Structures Discrètes et de Didactique at IMAG (CNRS-UJF) in 1988. Both DOS and Macintosh versions of this code are now distributed by Brooks/Cole Publishing Company. A demo version is available at the Washington University archive: wuarchive.wustl.edu.

*The Geometer's Sketchpad* is available from Key Curriculum Press. The original version of *The Geometer's Sketchpad*, introduced in 1991 for the Macintosh, has enjoyed a more widespread distribution in North America than *Cabri-Géomètre.* A Windows version of *The Geometer's Sketchpad* was introduced in March 1993.

This review compares *Cabri-Géomètre* version 2.1 and *The Geometer's Sketchpad* version 2.0, both running on a Macintosh LCII with System 7.

## Features Found in Both Systems, Contrasted

(1) Creating a Locus of Points

Both *Cabri* and *Sketchpad* allow the user to modify a sketch while the locus of a chosen point is marked. Neither allows dynamic modification or preservation of the locus. *Cabri* does provide an option to draw automatically the locus in a continuous manner. See Figure 1 for a comparison of the loci from *Cabri* and *Sketchpad.* This figure also illustrates one use of the feature, showing the geometric construction of conic sections.

(2) Macros

The sole purpose of macros in *Cabri* is to minimize work. They allow multi-step operations to be condensed to a single menu command. By identifying the initial and final objects of an already-existing construction, *Cabri* will create the appropriate macro. *Sketchpad* takes a more algorithmic approach—the user must first declare the intention to create a macro and then record the construction step-by-step. *Sketchpad* macros can call themselves and therefore create recursive constructions; *Cabri* does not have this option.

(3) Measurements and Calculations

In *Cabri*, angle measures and segment lengths can be superimposed on the figure. They remain positioned near the object and change dynamically. These measurements, as well as areas of circles and polygons, can be reported in table form. However, no *true* arithmetic calculations can be performed by *Cabri*. *Sketchpad* reports measurements and calculations as data whose locations are independent of their objects. *Sketchpad* also provides a pull-down scientific calculator, which can import previous measurements and calculations in a dynamic manner.

(4) Comments

Both systems allow the user to attach comments to sketches. *Cabri* creates a separate window for comments and hence requires window rescaling to view both a sketch and its comments simultaneously. It should be noted, however, that when a sketch is printed, the comments are printed with the sketch. *Sketchpad* permits text to be placed directly on the sketch. Thus, comments can be placed and printed at any location.

(5) Transformations

*Cabri* allows the user to reflect a point across a line. From this basic operation, macros can be built to perform other transformational geometry constructions, but they quickly become complicated. *Sketchpad* provides a built-in selection of such transformational constructions as translations, reflections, rotations, and dilations that are easy to use.

(6) Order of Object Selection

In the *Cabri* environment, commands are selected prior to the selection of objects. However, *Sketchpad* requires the opposite order of selection.

## Features Distinctive to *Cabri-Géomètre*

(1) Check for Invariance

A principal feature of *Cabri* is its ability to check for "apparent" invariants within a geometric diagram. Properties which can be checked are collinearity (alignment), membership of a point on a line, parallelism, perpendicularity, and equality of segment lengths. *Cabri* apparently uses visual tests, rather than logical deduction, to choose one of the following responses:

(a) This property is not true.
(b) This property is true in general.
(c) This property looks true in this case of the figure but is false in general.
(d) This property is true for the figure, but *Cabri* cannot determine whether it is true in general.

If the property is true for the figure as drawn but false in general, *Cabri* will provide a counterexample. For example, Figure 2 illustrates a specific example where the angle bisector of a triangle appears to be perpendicular to the opposite side. This property is not true in all cases and *Cabri* will offer a counterexample if requested.

*Cabri* almost always deduces the appropriate conclusion, though we found that it could sporadically report responses (b) and (d) for the same construction.

We found the ability to check properties the most attractive feature of *Cabri*. For the learning and discovery of geometry, this ability allows students to notice patterns and test conjectures. With minor debugging, annoying errors will hopefully be minimized.

(2) Redefinition of objects

In a DGS, it is necessary for the software to make distinctions concerning the relationships among the various objects, e.g. points on objects vs. points of intersection vs. free-moving points vs. midpoints vs. circle centers. At times, it is convenient to revise these relationships; *Cabri* allows such changes.

(3) Customized Construction Menu

*Cabri* permits menus to be customized so that particular commands are hidden. This allows for a structured development in the classroom. For example, the *Perpendicular bisector* construction command could be removed from the menu until students are familiar with the step-by-step process.

## Features Distinctive to *The Geometer's Sketchpad*

(1) Action Buttons

*Sketchpad* allows the user to create on-screen buttons to perform an action on command. Two of the most useful action buttons are *Animate* and *Sound.*

Dragging points to make dynamic changes is an essential feature for a DGS. *Sketchpad* will simulate the dragging of a point by automatically moving it along a circle or line segment, thus animating the figure. For example, Figure 3 shows the creation of a sine curve by animation.

In addition to adding text as comments on a sketch, the user can record sounds and leave them as buttons on the sketch to be played at any time. These sounds could be used as comments, suggestions, or whatever creative idea a student or teacher may come up with.

(2) Sharing Capabilities

If *Sketchpad* is being used in a Macintosh lab that is networked with AppleTalk, sketches may be transported freely from station to station.

Although *Cabri-Géomètre* and *The Geometer's Sketchpad* have similar features, we have highlighted key differences among the packages. As new DGS's are produced and marketed, manufacturers will assimilate ideas from existing software to develop systems even more powerful than those now available.

## REFERENCES

[1] National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics*, NCTM, Reston, Virginia, 1989.

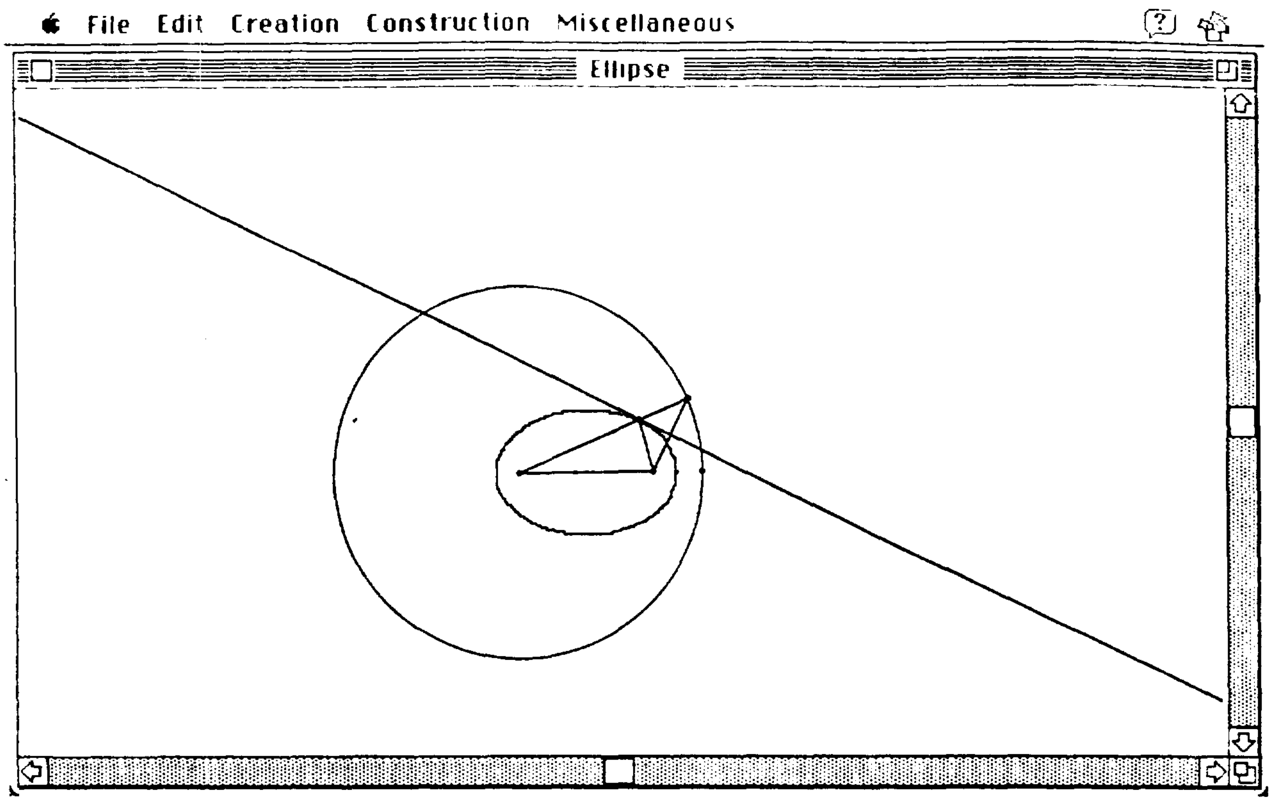



Figure 1a. An ellipse created as the locus of a point by *Cabri-Géomètre*.

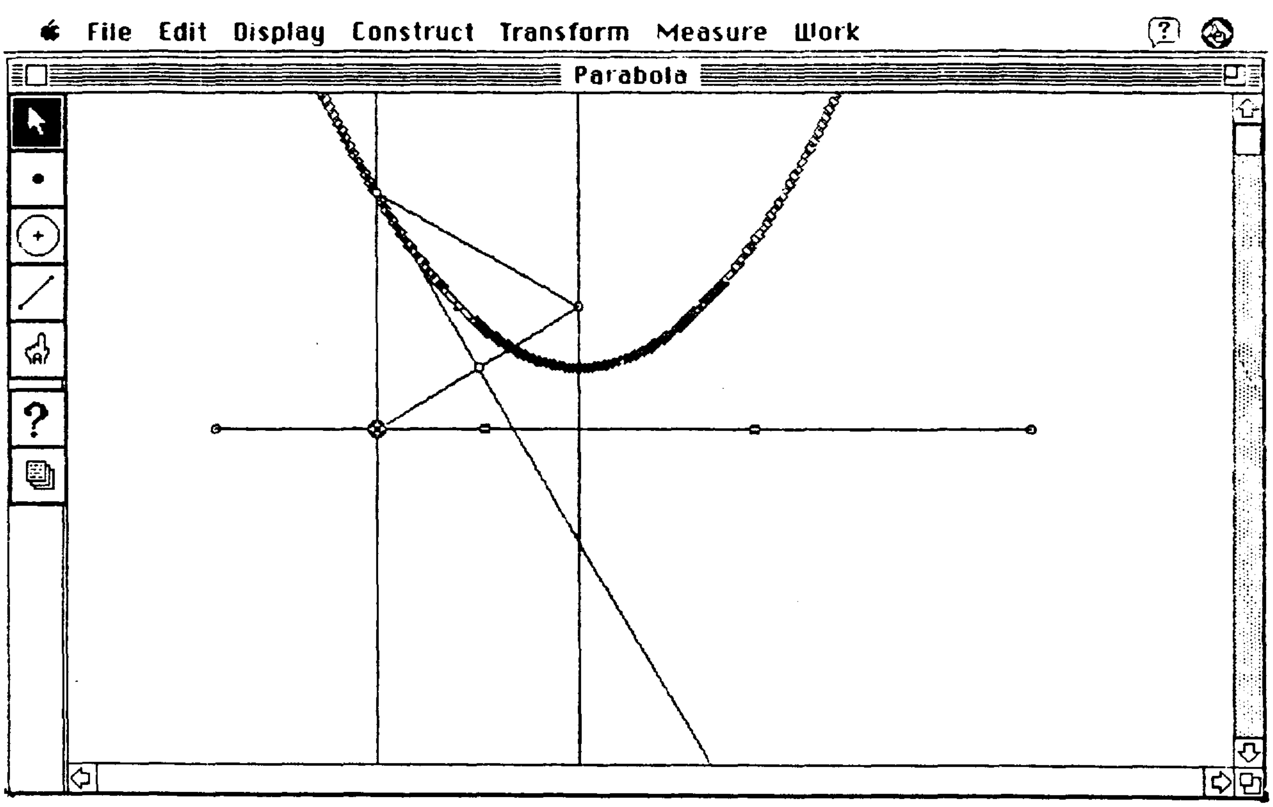



Figure 1b. A parabola created as the locus of a point by *Geometer's Sketchpad*.

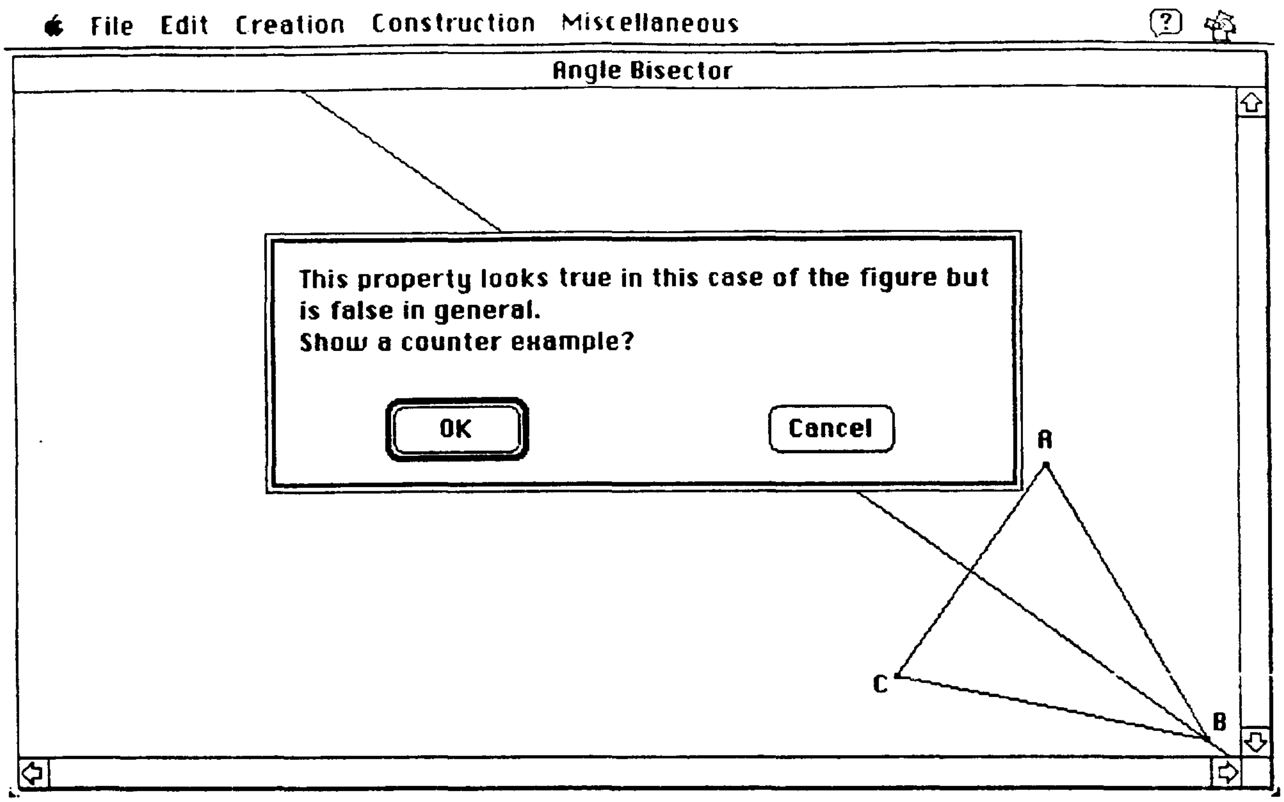



Figure 2. The response of *Cabri-Géomètre* after checking for perpendicularity of segment *AC* with the bisector of angle *ABC*.

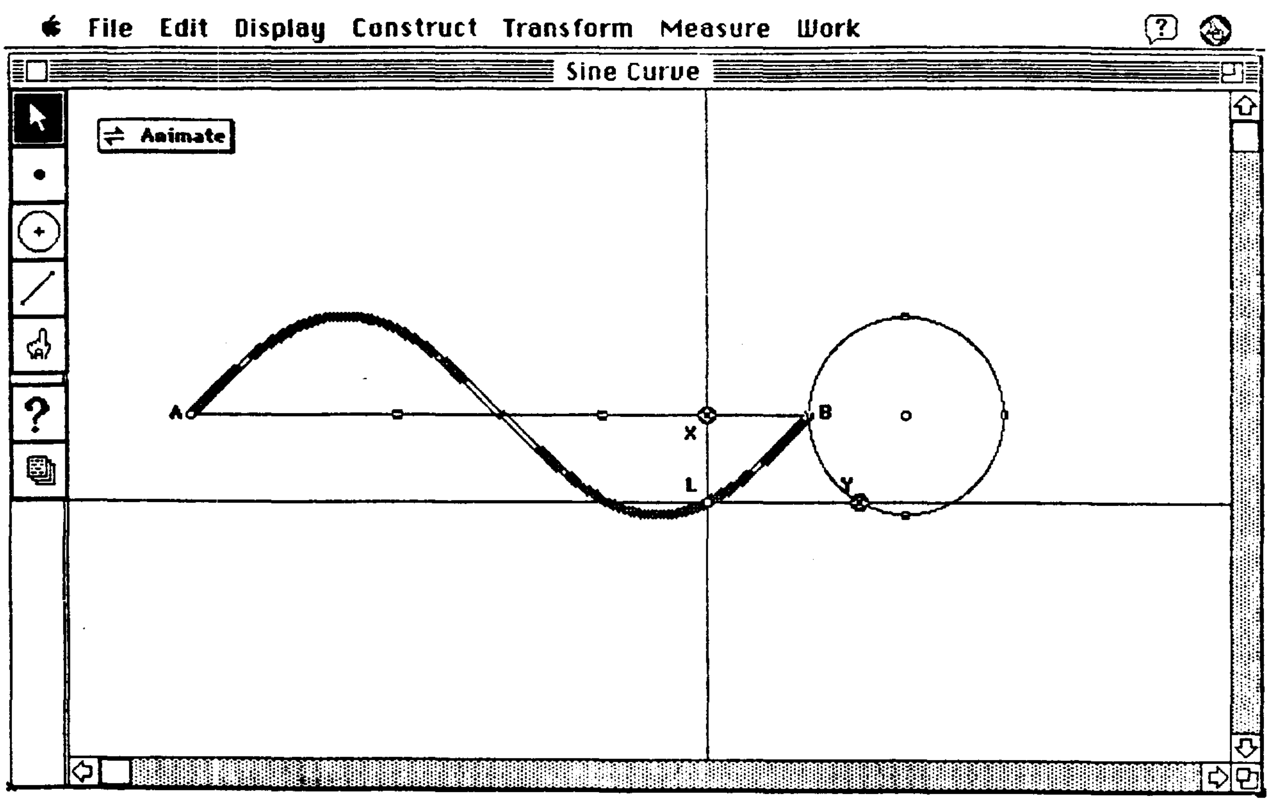



Figure 3. This action button on the *Geometer's Sketchpad* will animate point *X* along the segment *AB* and *Y* along the circle. By tracing the locus of L, the sine curve is constructed.

# *Appendix C*

# *Bibliography*

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